MATH 551 - Midterm Correction

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October 6, 2014

My original score was a 21/40. I received a 1/10 for problem 4 (I ran out of time).

4.

a) Because A, X, B are collinear, we know that $\exists \alpha_1 \mid AX = \alpha_1 AB$, and similarly $\exists \alpha_2$ for AY and AC ($AY = \alpha_2 AC$). We are given that $XY || BC \Rightarrow \exists \mu \mid \overrightarrow{XY} = \mu \overrightarrow{BC}$. We may write

$$AY - AX = \alpha_2 AC - \alpha_1 AB = \mu(\overrightarrow{AC} - \overrightarrow{AB}) \Rightarrow \mu\overrightarrow{AC} - \mu\overrightarrow{AB} - \alpha_2 AC + \alpha_1 AB$$

and by writing these vectors in terms of their endpoints (e.g. $\overrightarrow{AB} = B - A$) and solving for 0 we achieve

$$0 = \mu\overrightarrow{C} - \mu\overrightarrow{A} - \mu\overrightarrow{B} + \mu\overrightarrow{A} - \alpha_2\overrightarrow{C} + \alpha_2\overrightarrow{A} + \alpha_1\overrightarrow{B} - \alpha_1\overrightarrow{A}$$

canceling and combining terms we come to

$$0 = (\alpha_2 - \alpha_1)\overrightarrow{A} + (\alpha_1 - \mu)\overrightarrow{B} + (\mu - \alpha_2)\overrightarrow{C}$$

Now we notice that the sum of the coefficients of $\overrightarrow{A}, \overrightarrow{B}, \overrightarrow{C}$ is zero, and by theorem 2 we know that if this equation is zero (which it is) and the sum of the coefficients is zero (which it is) and the coefficients are not all zero, then we must have that $\overrightarrow{A}, \overrightarrow{B}, \overrightarrow{C}$ are collinear. But we know that $\overrightarrow{A}, \overrightarrow{B}, \overrightarrow{C}$ are not collinear because they form a triangle, so one of our assumptions must have been wrong. We therefore conclude that each of the coefficients must be zero, and therefore we find that $(\alpha_2 - \alpha_1) = 0 \Rightarrow \alpha_2 = \alpha_1$. Thus we have shown $\exists \alpha \mid AX = \alpha AB$ and $AY = \alpha AC$.

b) By part a) we know that $\exists \alpha \mid \overrightarrow{AU} = \alpha\overrightarrow{AB}$ and $\overrightarrow{AV} = \alpha\overrightarrow{AD}$. $\overrightarrow{AW}$ can be written as $\overrightarrow{AU} + \overrightarrow{AV}$, but using our results from part a) we know $\overrightarrow{AW} = \overrightarrow{AU} + \overrightarrow{AV}$ $= \alpha(\overrightarrow{AB} + \overrightarrow{AD})$, but we notice $\overrightarrow{AB} + \overrightarrow{AD}$ is exactly $\overrightarrow{AC}$, and so we must have it that $\overrightarrow{AW} = \alpha\overrightarrow{AC}$. And because these vectors are scalar multiples of each other, we know that their endpoints must reside on a single line and thus we may conclude A, W, C are collinear.